

An hybrid multiscale model for immersed granular flows

Matthieu Constant^{1,*}, Frédéric Dubois^{2,3,**}, Jonathan Lambrechts^{1,***}, and Vincent Legat^{1,****}

¹ Université catholique de Louvain, Avenue Georges Lemaître 4-6 bte L4.05.02, 1348, Louvain-la-Neuve, Belgique

² Laboratoire de Mécanique et Génie Civil - CNRS/UM, Montpellier, France

³ Laboratoire de Micromécanique et Intégrité des Structures - CNRS/UM/IRSN, France.

Abstract. The intensive use of immersed granular flows has motivated a large amount of work in this field of research. We present here a new numerical model to represent accurately and efficiently mixtures of fluid with grains. On one hand, the motion of the grains is solved at the grain scale by a contact dynamics method that consider grains as discrete elements. On the other hand, the fluid flow is computed from a continuous model of the mixture at a greater scale. The link between the two scales is provided by an interaction force based on the Darcy's law for porous media and used as a closure relation in the equations of the model. Some results of simulations in a two dimensional space are provided to prove the efficiency of the implementation.

1 Introduction

A lot of numerical methods have been developed to understand and predict immersed granular flows i.e. mixture of grains in suspension within a fluid [1]. We can separate the different types of physical models for immersed granular flows with respect to the scale at which the flow is modeled [2]. At large scale, the grains are introduced in the fluid in an implicit way. On one hand, some models consider the medium as a mixture described as a non newtonian fluid [3]. On the other hand, the grains can be introduced via a dissipative force of friction [4]. This force is used as a closure relation in the momentum equations and depends largely on the considered problem [5]. At a smaller scale, direct methods consider the two phases distinctly. The dynamics of the fluid is solved in an Eulerian way while the motion of the grains is solved in a Lagrangian way. This can be numerically done by remeshing the fluid phase at each time step [6]. Other approaches use methods of penalty or Lagrange multipliers to avoid the numerical cost imposed by the successive creations of a mesh [7]. Finally, a Lattice-Boltzmann approach can be used to simulate immersed granular flows without a mesh [8].

Discrete elements methods (DEM) are commonly used to describe trajectories of grains in a Lagrangian way for dry granular flows [9]. Specific models are able to solve grain-grain and grain-boundary collisions. We can distinguish several types of DEM. Collision methods solve contacts sequentially. They are not applicable to dense granular media where the time between two collisions is very short. Other methods solve all the contacts that happened

during a given time step. They are based on two models of grains that change the representation of the contacts. The *smooth grain* model allows a slight deformation or an interpenetration of the grains. The elastic, plastic and friction forces between the grains are deduced from these deformations/interpenetrations. On the contrary, the *nonsmooth grain* model totally bans such deformations. An implicit resolution of the movement equations of the grains and the contact forces is needed for this method.

In this paper, we present an hybrid method [10] that can be viewed as a compromise between direct methods and methods based on continuum mechanics. The motion and contacts of grains are solved using a DEM at the grain scale while the fluid flow is obtained from a continuous description of the mixture at a greater scale using finite elements method. It is then possible to solve problem faster while describing accurately the physics of grains.

2 Model

Our model is based on a well-known hypothesis of the continuum mechanics: the separation of scales. The fluid flow characteristics are deduced from a continuous model of the mixture at a larger scale than the grains. The incompressible Navier-Stokes equations are averaged on elementary subsets of the domain. These elementary subsets are the sum of a fluid volume Ω_f and a solid volume Ω_p . The size of the grains have to be negligible in comparison with the size of the subsets.

Let us define the porosity that corresponds to the volume fraction of the fluid in an elementary subset:

$$\phi = \frac{\Omega_f}{\Omega}$$

*e-mail: matthieu.constant@uclouvain.be

**e-mail: frederic.dubois@umontpellier.fr

***e-mail: jonathan.lambrechts@uclouvain.be

****e-mail: vincent.legat@uclouvain.be

The averaged Navier-Stokes equations on an elementary subset are:

$$\frac{1}{\Omega} \int_{\Omega_f} \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} \right) d\Omega = 0$$

$$\frac{1}{\Omega} \int_{\Omega_f} \left(\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) - \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} \right) d\Omega = 0$$

where ρ is the density of the fluid, $\boldsymbol{\sigma}$ is the Cauchy tensor, \mathbf{v} is the velocity and \mathbf{g} is the gravity. The intrinsic fluid volume average of the velocity on an elementary subset is defined by:

$$\langle \mathbf{v} \rangle_f = \frac{1}{\Omega_f} \int_{\Omega_f} \mathbf{v} d\Omega$$

The velocity is then splitted in this average and a perturbation with zero mean value:

$$\mathbf{v} = \langle \mathbf{v} \rangle_f + \mathbf{v}'$$

The intrinsic solid volume average operator can be defined in a similar way. Then, using some average theorems [11, 12], we derive the average form of the Navier-Stokes equations:

$$0 = \nabla \cdot [\phi \langle \mathbf{v} \rangle_f + (1 - \phi) \langle \mathbf{v} \rangle_p]$$

$$0 = \rho \frac{\partial (\phi \langle \mathbf{v} \rangle_f)}{\partial t} + \rho \nabla \cdot (\phi \langle \mathbf{v} \rangle_f \langle \mathbf{v} \rangle_f) + \phi \nabla \cdot \langle \mathbf{p} \rangle_f - \phi \rho \mathbf{g}$$

$$- \mu \nabla^2 (\phi \langle \mathbf{v} \rangle_f) + \mu \frac{\nabla (\phi \langle \mathbf{v} \rangle_f) \cdot \nabla \phi}{\phi} - \mu \langle \mathbf{v} \rangle_f \frac{\nabla \phi \cdot \nabla \phi}{\phi}$$

$$- \underbrace{\nabla \cdot \left(\frac{\mu}{\Omega} \int_s \mathbf{n} \mathbf{v} ds - \rho \phi \langle \mathbf{v}' \mathbf{v}' \rangle_f \right) - \frac{1}{\Omega} \int_s (\mu \mathbf{n} \cdot \nabla \mathbf{v}' - \mathbf{n} p') ds}_F$$

where μ is the dynamic viscosity, \mathbf{n} is the outward normal to the interface s of the two phases.

Finally, this mathematical model is closed as we replace those last terms by a force \mathbf{F} that integrates the interactions of the fluid with all the grains in the representative volume. The interaction force between a grain i and the fluid can be written at the grain scale:

$$\mathbf{F}_i = -V_{p_i} \nabla \cdot \langle \mathbf{p} \rangle_f - \mathbf{D}_i + \mathbf{g} V_{p_i} \frac{\rho_{p_i} - \rho}{\rho_{p_i}}$$

where \mathbf{D}_i is the drag force, V_{p_i} is the volume of the grain i and ρ_{p_i} is its density. In our implementation, if the mixture is assumed to be composed of identical grains, we can use the formula given by Di Felice and Rotondi [13]

$$\mathbf{D}_i = f(\phi) C_d(\text{Re}_{p_i}) \frac{A_{p_i} \rho}{2} |\mathbf{v}_{p_i} - \langle \mathbf{v} \rangle_f| (\mathbf{v}_{p_i} - \langle \mathbf{v} \rangle_f)$$

where A_{p_i} is the projected area of the grain on a plane perpendicular to the grain motion and Re_{p_i} is the grain Reynolds number:

$$\text{Re}_{p_i} = \frac{2r\rho\phi}{\mu} |\mathbf{v}_{p_i} - \langle \mathbf{v} \rangle_f|$$

To take into account the effect of the neighbor grains, we use a function $f(\phi)$ given by Wen and Yu [14]:

$$f(\phi) = \phi^{-1.8}$$

while the drag coefficient $C_d(\text{Re}_{p_i})$ is provided by Dallavalle et al. [15]

$$C_d(\text{Re}_{p_i}) = \left(0.63 + \frac{4.8}{\text{Re}_{p_i}^{0.5}} \right)^2$$

3 Results

Since the theoretical researches conducted by Brinkman [17] in 1947 on the fluid flow in an immersed swarm of grains, a lot of experiments have been realized to understand the observed micro- and macroscopic behaviors. An overview of these results can be found in [16, 18].

Just after being introduced in the fluid, the swarm falls and some grains escape from the closed volume of the swarm at the rear of the motion. They form a sort of tail while the lower part of the swarm forms something like a helmet giving to the whole grains the form of a mushroom. Twenty years later, Nitsche and Batchelor [19] performed numerical simulations to better understand this leakage of grains. They characterized the trajectories of the grains inside the swarm by decomposing the dynamics in two types of kinematics: a circular motion due to the toroidal velocity field inside the swarm and a random motion due to the hydrodynamical interactions. The leakage is then produced by the random motions that lead the grains outside the swarm where the streamlines are not closed.

The grains in the tail slowly fall because they are no longer dragged by the swarm [16]. The tail breaks away from the hemispherical part. Then, the center of the swarm is almost empty but the fluid can't cross it because of the overpressure in front of the motion. This overpressure point change into a ring below the swarm that becomes a torus and the fluid will then be able to cross it [18]. The torus becomes unstable and divides into two new smaller swarms that can repeat the same process if they are made with enough grains.

Simulations in two dimensions are performed with our model to compare the results with the simulations and experiments of Machu et al. [16]. Figure 1 shows the results for glass beads falling in glycerin. We clearly see the different steps corresponding to the experimental observations. Initially, we release a spherical swarm (circular in 2D) in the fluid. In figure 1a, the leakage of grains at the rear of the motion leads to the formation of a reverse mushroom. The tail we obtain in simulations is smaller than the one shown by Machu et al. [16] but it could be explained by comparing the number of grains in our swarm. In two dimensions, the number of grains is much less than in three dimensions. The computed velocity field looks like a cross-section of the toroidal field described by Metzger et al. [18]. The situation in Figure 1g shows the tail is no longer attached to the swarm. The overpressure point also divides in two part that can be again considered as a cross section of the ring described above (see Figure 1i).

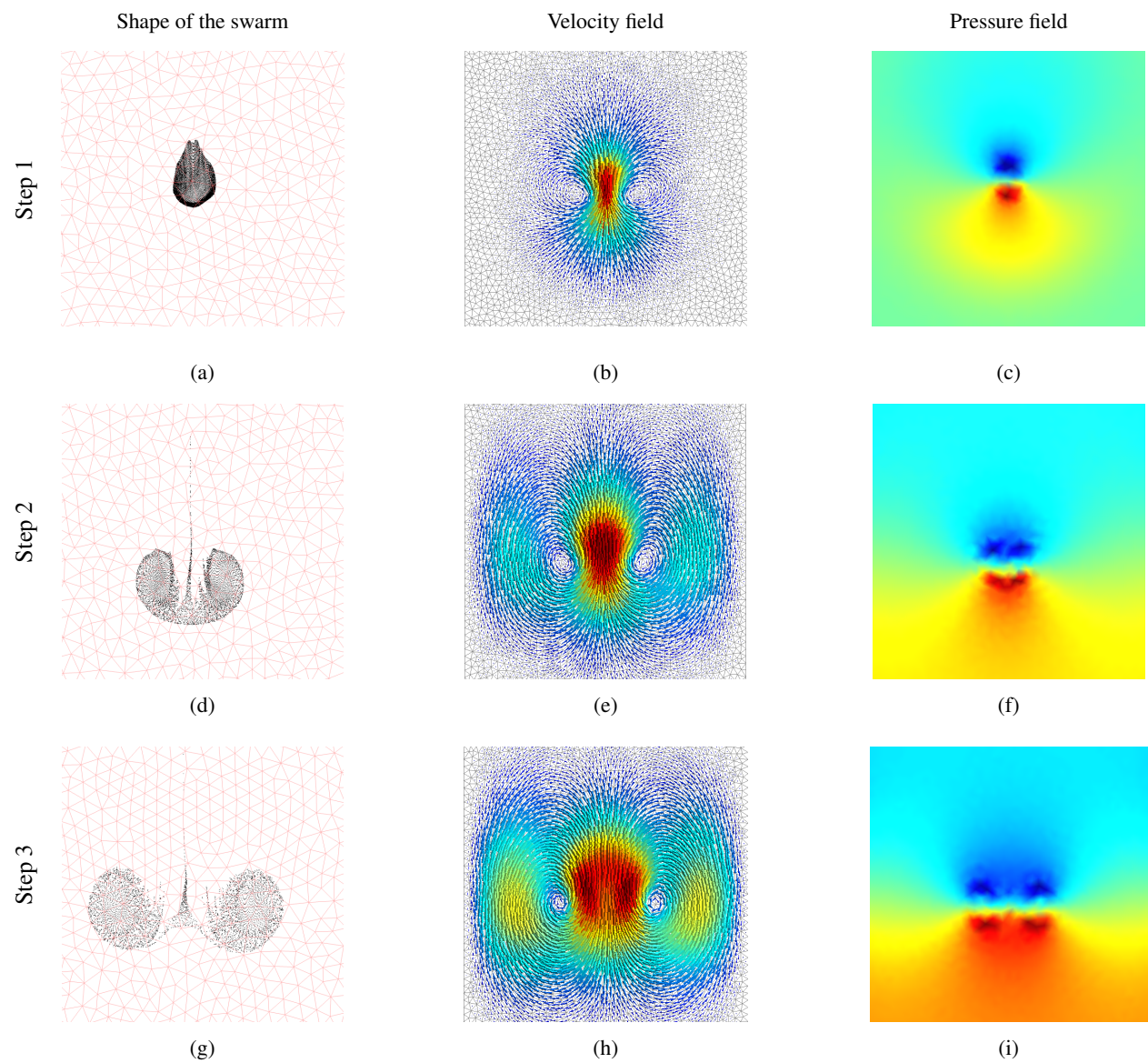


Figure 1: Evolution of a swarm made up of glass beads falling in glycerin

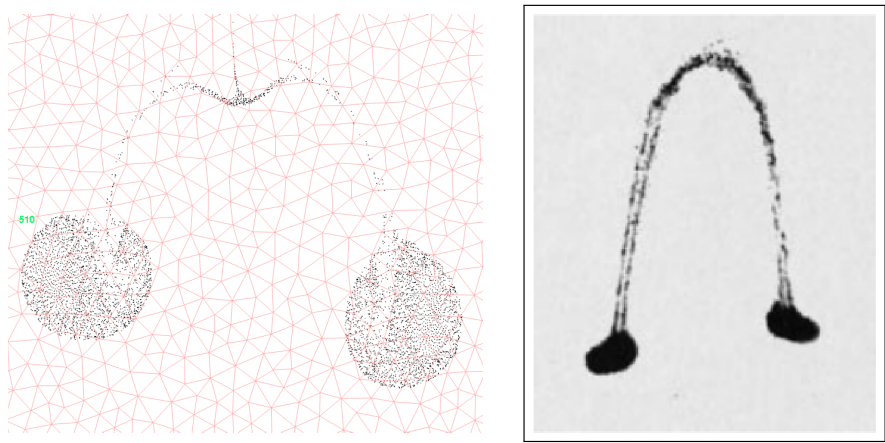


Figure 2: Division of the swarm (comparison with the experiment of Machu et al. [16])

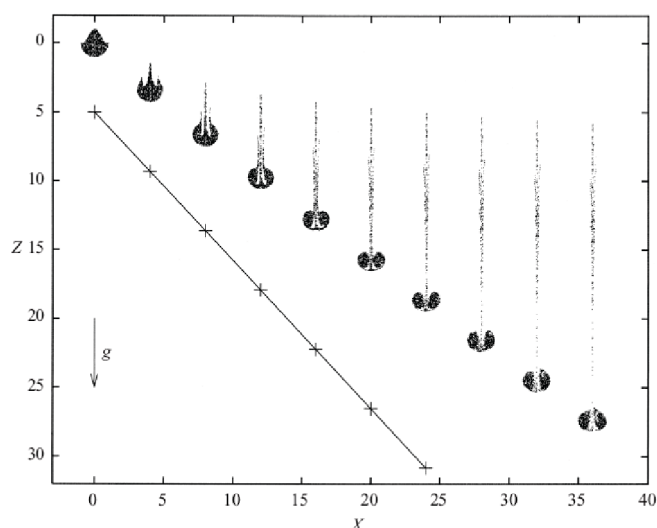


Figure 3: Simulation of Machu et al. [16]

The division of the swarm in two new smaller swarms is shown in Figure 2. Finally, it appears that the numerical simulation provides a quite similar behaviour than the results of Figure 3.

4 Conclusion

We develop an hybrid multiscale model to compute immersed granular flows. This model solves the contacts between grains with a DEM. We consider a non-smooth contact dynamics method and inelastic collisions. The fluid is solved at a greater scale by averaging the Navier-Stokes equations on an elementary subset containing the mixture of grains within the fluid. These averaged equations are solved by a finite elements method. With this model, we can compute a wide range of immersed granular flows from pure fluids to porous media with a reasonable computational time while the physics of the grains is described accurately.

An interaction force between grains and the fluid link the two scales of the model. This force is introduced in the momentum equations to replace some perturbation and interaction terms appeared during the averaging. This interaction force depends on the drag force that is determined empirically. The Rotondi and Di Felice's law neglects some important characteristics of the mixture such as the diversity of the grains encountered in real flows. More complex formula can be used to incorporate those effects.

Those preliminary results are very promising and demonstrate the qualities of the model. Firstly, we are able to compute the forces and constraints applied on each grain and their macroscopic motions can be compared

with some simulations and experiments found in literature. Secondly, we can represent the flow induced by the displacements of the grains that can't be obtained in real experiments. And finally, we have shown that we could obtain accurate results using a representation of the fluid at a greater scale than the grains. Of course this is just a preliminary test of our model and other simulations should be achieved to show the real benefit of the contacts representation. The next step consists of generalize our approach in three dimensions in order to obtain better quantitative comparisons with experiments.

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